

University of California, Berkeley
Physics 110A, Section 2, Spring 2003 (*Strovink*)

PROBLEM SET 2

1.
Prove that

$$\frac{d}{dx}\theta(x-x') = \delta(x-x') ,$$

where δ is a Dirac delta function and, as usual, $\theta(x-x') = 0$ (1) for $x < x'$ ($x > x'$). That is, prove that

$$\int_{-\infty}^{\infty} dx f(x) \frac{d}{dx}\theta(x-x') = f(x')$$

for any differentiable function f . [Hint: Integrate by parts.]

2.

Because the magnetic field is divergenceless ($\nabla \cdot \vec{B} = 0$), without loss of generality it can be written as the curl of a *vector potential* \vec{A} : $\vec{B} = \nabla \times \vec{A}$. Provided that the scalar potential V is varied as well, some variation is possible in the definition of \vec{A} ; not quite standardly, one may require $\nabla \cdot \vec{A} = 0$ (this is called “Coulomb gauge”). Work this problem in Coulomb gauge.

(a.)

Using Ampère’s Law in the steady state, $\nabla \times \vec{B} = \mu_0 \vec{J}$, show that

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} .$$

(b.)

Using the Green function for the differential operator ∇^2 , derive the steady-state integral-equation solution for \vec{A} :

$$\frac{4\pi}{\mu_0} \vec{A}(\vec{r}) = \int d\tau' \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} ,$$

where the integral is taken over all space.

3.+4.

According to the Proca equations (a relativistically invariant linear generalization of Maxwell’s

equations accommodating the possibility of a finite rest mass m_0 for the photon), Gauss’s law is modified to become

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \frac{\phi}{\bar{\lambda}^2} , \quad (1)$$

where ϕ is the electrostatic potential and

$$\bar{\lambda} \equiv \frac{\hbar}{m_0 c}$$

is the reduced (by 2π) Compton wavelength of the photon.

Following Williams, Faller, and Hill, *Phys. Rev. Lett.* **26**, 721 (1971), consider two concentric spherical perfectly conducting shells of radii R_1 and R_2 , respectively, with $R_2 > R_1$. Imagine that the inner sphere is isolated and that the outer shell is driven by an RF oscillator so that it has a potential (relative to ∞)

$$V_2(t) = V_0 \cos \omega t .$$

(Strictly speaking, electrostatic potential is undefined in a time-dependent problem. However, as long as the geometry remains spherically symmetric, \vec{E} will remain curlless and V can still be defined.)

If it is nonzero at all, the last term in Eq. (1) is very small. To obtain an approximate solution to Eq. (1), we take advantage of this fact by using the *method of perturbations*.

(a.)

If the last term in Eq. (1) were zero, what would be the unperturbed solution $\phi_u(r, t)$ for the potential anywhere inside the outer shell?

(b.) Approximate $\phi \approx \phi_u$ in Eq. (1) (any error is second order in the small quantity $\bar{\lambda}^{-2}$). Consider a spherical surface at radius r , where $R_1 < r < R_2$. Take the volume integral of both sides of Eq. (1) within the radius r . Using the Divergence Theorem, convert the left-hand side into a surface integral of \vec{E} and express the result in terms of $E_r(r, t)$. Assuming that there is no charge on the surface of the inner sphere, do the

volume integral on the right-hand side to obtain a result proportional to $\bar{\lambda}^{-2}$. Solve for $E_r(r, t)$.

(c.)

Integrate $E_r(r, t)$ between R_1 and R_2 to obtain $v(t)$, the voltage difference between R_2 and R_1 , in terms of $\bar{\lambda}^{-2}$.

(d.)

Suppose $R_1 = 0.5\text{m}$ and $R_2 = 1.5\text{m}$. If the amplitude of $v(t)$ were measured to be $10^{-15}V_0$ (such a signal would easily have been detected by Williams *et al.*), what would be the photon mass m_0 ? Express m_0 in eV/c^2 , and also as a ratio to the $91\text{ GeV}/c^2$ mass of the heavy photon Z^0 discovered in 1981.

where k is a constant. Show that the total charge that created this potential is zero. [*Hint*: Is the charge density finite at the origin?]

5.

A hollow spherical shell carries volume charge density

$$\rho(r) = \frac{k}{r^2}$$

in the region $a < r < b$, where k is a constant. If the electrostatic potential $V = 0$ at ∞ , find V at the origin. Do this

(a.)

By using Gauss's law and spherical symmetry to evaluate \vec{E} everywhere, then line-integrating \vec{E} from ∞ to the origin.

(b.)

By using Griffiths Eq. (2.29).

6.

Consider a thin uniformly charged disk. At a point on the cylindrical symmetry axis above the disk, show that the electric field is proportional to the solid angle subtended by the disk.

7.

Consider a solid cylinder of length l and radius $b \ll l$, coaxial with the z axis and centered at the origin. The cylinder carries a uniform volume charge density ρ_0 . On the z axis at a height z , where $|z| < l/4$, estimate the electric field. Make any approximations that you find sensible and convenient.

8.

Consider the "screened Coulomb potential"

$$V(\vec{r}) \propto \frac{e^{-kr}}{r},$$